REMARKS ON THE FREQUENCY SHIFT IN
HIGH INTENSITY COMPTON SCATTERING*

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Various papers have been published recently in which the radiation interaction of a free electron subject to an intense monochromatic photon field have been discussed (Ref. 1). According to these calculations light will be generated at all harmonics of the fundamental frequency of the primary radiation field (harmonic generation). However there is one point which needs clarification. Whereas some authors claim that the scattered photons exhibit an intensity-dependent characteristic frequency shift (Brown, Kibble and Goldman) others claim that they do not (Fried, Eberly and von Roos). The fact that the frequency shift has been found by those authors who were using a semi-classical approximation i.e., an approximation in which the electron is subjected to a <u>given</u> external electromagnetic wave is all the more surprising since in such calculation a reaction of the electron back on the incident beam is completely excluded and therefore the origin of the frequency shift is difficult to understand. In the following we will re-examine the question using again a semi-classical approach. However, we will show that by using wave functions with correct initial conditions there will be no frequency shift, in agreement with Fried, Eberly and von Roos.

*This paper presents results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NAS7-100, sponsored by the National Aeronautics and Space Administration.

Since the frequency shift in question is independent of any spin effects, we choose for simplicity the Klein Gordon equation to describe the motion of an electron in a given external electromagnetic field. If the vector potential of the monochromatic plane wave, which we take to be linearly polarized (in the x direction) and traveling in the z direction, is given by

$$\vec{A} = \frac{mc^2}{e} \alpha \, \vec{e}_x \cos k_o(z - ct) \tag{1}$$

with

$$\alpha^2 = \frac{4\pi \, e^2 \, \hbar \bar{n}}{m^2 \, c^2 \, \omega_o} \tag{2}$$

a dimensionless parameter measuring the strength of the external radiation field ($\bar{n}$ is the number of photons per cm^3), we have as equation of motion for the electron

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \psi = \left\{ \left(\frac{mc\alpha}{\hbar} \right)^2 \cos^2 k_o(z - ct) + \left(\frac{mc}{\hbar} \right)^2 + 2i \, \frac{mc\alpha}{\hbar} \cos k_o(z-ct) \frac{\partial}{\partial x} \right\} \psi \tag{3}$$

In the absence of the electromagnetic field (i.e., putting $\alpha = 0$) Eq. (3) has the well known solution

$$\psi = \psi_o = e^{i\vec{k}\cdot\vec{r} - i\Omega t} \tag{4}$$

with

$$\Omega = c \left[\left(\frac{mc}{\hbar} \right)^2 + k^2 \right]^{\frac{1}{2}} \tag{5}$$

[redacted]

In the presence of the electromagnetic beam we put:

$$\psi = \psi_o \, \zeta(z - ct, t) \tag{6}$$

for the solution of Eq. (3). The ansatz (6) is quite different from that adopted by former authors. In particular, Brown and Kibble as well as Goldman (Ref. 1) choose a wave function of the form

$$\psi \sim \psi_o \, \zeta(z - ct) \tag{7}$$

in accordance with the old Volkov solution of the Dirac equation (Ref. 2). However an ansatz of the type (7) is incapable of describing initial conditions. This is the crux of the matter. Because physically the electromagnetic field (1) is created at some initial time T, say. That means that the electron was free prior to T or

$$\zeta = 1 \quad \text{for } t \leq T \tag{8}$$

a condition impossible to satisfy with ansatz (7). In the following we will show, using the ansatz (6) with proper initial conditions, that there will be no frequency shift in accordance with the classical calculation by Vachaspati (Ref. 3).

Introducing, then, the abbreviation

$$u = z - ct \tag{9}$$

and inserting the ansatz (6) into Eq. (3) yields:

$$- \frac{1}{c^2} \frac{\partial^2 \zeta}{\partial t^2} + \frac{2}{c} \frac{\partial^2 \zeta}{\partial t \, \partial u} + 2i \frac{\Omega}{c^2} \frac{\partial \zeta}{\partial t} + 2i\left(k_z - \frac{\Omega}{c}\right) \frac{\partial \zeta}{\partial u} = \left\{ \left(\frac{mc\alpha}{\hbar}\right)^2 \cos^2 k_o u - 2k_x \frac{mc\alpha}{\hbar} \cos k_o u \right\} \zeta(u, t) \tag{10}$$

As initial condition we adopt:

$$\zeta(z, 0) = 1 \tag{11}$$

i.e., the laser beam (1) is switched on at $t = 0$. In order that Eq. (3) be valid throughout and that there be no extraneous singular potentials, we must also have:

$$\frac{\partial \zeta}{\partial t} - c\,\frac{\partial \zeta}{\partial u} = 0 \qquad \text{at } t = 0 \tag{12}$$

meaning that the first time-derivative of the wave function (6) is continuous at $t = 0$. Eq. (10) is solved with the expression:

$$\zeta = \sum_{n=-\infty}^{+\infty} e^{ink_o u}\, F_n(t) \tag{13}$$

provided that the F_n satisfy:

$$\frac{d^2 F_n}{dt^2} - 2i(\Omega + n\,c\,k_o)\,\frac{d F_n}{dt} + 2n\,c^2\,k_o\left(k_z - \frac{\Omega}{c}\right) F_n$$

$$= \frac{mc^3\alpha}{\hbar}\,k_x\left(F_{n-1} + F_{n+1}\right) - \left(\frac{mc^2\alpha}{2\hbar}\right)^2 \left(F_{n-2} + 2F_n + F_{n+2}\right) \tag{14}$$

The initial conditions are satisfied if:

$$\left.\begin{aligned} F_n(0) &= \delta_{no} \\ F_n'(0) &= 0 \end{aligned}\right\} \tag{15}$$

The system, Eqs. (14) and (15), determines the wave function (6) uniquely. The solution of Eq. (14) is quite complex and cannot readily be given in closed form. But since even for very high intensity laser beams α is small compared to one, a perturbation expansion in α is quite possible. From the structure of Eq. (14) it is easily seen that

$$F_n(t) = \alpha^n \left\{ G_n^o(t) + \alpha\, G_n'(t) + \alpha^2 G_n''(t)\, t \ldots \right\} \tag{16}$$

In 0^{th} order obviously:

$$F_n(t) = \delta_{no} \tag{17}$$

and in 1^{st} order:

$$F_o(t) = 1$$

$$F_{\pm 1}(t) = \frac{mck_x\alpha}{2k_o\left(k_z - \frac{\Omega}{c}\right)} \left[\left(1 - \frac{\alpha_\pm^{(1)}}{\alpha_\pm^{(2)}} \right)^{-1} \left\{ \pm e^{\alpha_\pm^{(1)} t} \mp e^{\alpha_\pm^{(2)} t} \frac{\alpha_\pm^{(1)}}{\alpha_\pm^{(2)}} \right\} \mp 1 \right] \tag{18}$$

with

$$\alpha_\pm^{(1)} = i(\Omega \pm ck_o) + \left(-(\Omega \pm ck_o)^2 \mp 2c^2 k_o (k_z - \frac{\Omega}{c}) \right)^{\frac{1}{2}} \tag{19}$$

$$\alpha_\pm^{(2)} = i(\Omega \pm ck_o) - \left(-(\Omega \pm ck_o)^2 \mp 2c^2 k_o (k_z - \frac{\Omega}{c}) \right)^{\frac{1}{2}} \tag{20}$$

and all other F_n vanish in this order. Further calculations along these lines will be given elsewhere. Here we will only show that the solution (6) with (13) does not lead to any frequency shift; the exact structure of the F_n is unimportant for this purpose. The interaction of an electron with the quantized transverse electromagnetic field is given by the Hamiltonian

$$H = \frac{ie\hbar}{mc}\left(\frac{2\pi\hbar c}{V}\right)^{\frac{1}{2}} \sum_{\alpha,\vec{K}} K^{-\frac{1}{2}}\, e^{(\alpha)}_{\vec{K}} \cdot \nabla\, e^{i\vec{K}\cdot\vec{r}} \left(a_{\alpha}(\vec{K}) + a^{+}_{\alpha}(-\vec{K})\right)$$

$$+ \frac{re^2\hbar}{mcV} \sum_{\alpha,\beta K,K'} (KK')^{-\frac{1}{2}}\, \vec{e}^{(\alpha)}_{K} \cdot \vec{e}^{(\beta)}_{K'}\, e^{i(\vec{K}+\vec{K}')\cdot\vec{r}} \left[a_{\alpha}(\vec{K})\right.$$

$$\left. + a^{+}_{\alpha}(-K)\right] \left[a_{\beta}(K') + a^{+}_{\beta}(-K')\right] \tag{21}$$

where the $a^{+}_{\alpha}(K)$, $a^{+}_{\alpha}(-K)$ are the usual photon creation and destruction operators and the $e^{(\alpha)}_{K}$ are unit transverse polarization vectors. The first-order matrix element for the emission of a photon of polarization γ and wave vector K is then

$$M(\gamma,\vec{K}) = \frac{e}{mc}\left(\frac{2\pi\ \hbar c}{VK}\right)^{\frac{1}{2}} \int_{0}^{t} d\tau \int d^3r\ \psi_{0}(\vec{k},\vec{r})^{*}\ \zeta(z - c\tau,\tau)^{*}$$

$$e^{-i\vec{K}\cdot\vec{r}}\ e^{(\alpha)}_{\vec{K}} \cdot \nabla\, \psi_{0}(\vec{k}',\vec{r})\ \zeta(z - c\tau,\tau) \tag{22}$$

There will also be a contribution from the second term of expression (21), as explained in detail elsewhere (Ref. 1, O. von Roos). This contribution is due to the fact that the A^2 term is capable of creating a photon in the mode γ, $\vec{K}$ and simultaneously adding or removing a photon from the beam. In any case, this contribution is omitted here for simplicity. The subsequent argument is unaffected by its omission.

Observing, then, Eqs. (22), (4) and (13) we find immediately that

$$M(\gamma,K) \sim \sum_{n,n'} \delta\left(-k_z + k_z' - K_z + (n - n')\, k_o\right) \delta(-k_x + k_x' - K_x)\, \delta(-k_y + k_y' - K_y) \qquad (23)$$

or in other words momentum is strictly conserved. There is no intensity dependent frequency shift.

REFERENCES

1. I. I. Goldman, Phys. Letters 8, 103 (1964); L. S. Brown and T. W. B. Kibble, Phys. Rev. 133, A705 (1964); Z. Fried and J. H. Eberly, Phys. Rev. 136, B871 (1964); O. von Roos, Phys. Rev. 135, A43 (1964); T. W. B. Kibble, Report at the Conference on Quantum Electrodynamics of High Intensity Photon Beams held at Durham, North Carolina on 26-27 August 1964 (unpublished).
2. D. M. Volkov, Z. Physik 94, 250 (1935).
3. Vachaspati, Phys. Rev. 128, 664 (1962) and 130, 2598 (1963).